# Deformation Dynamics of Radially Loaded Tubular Conductive Shell under High Pulsed Magnetic Field at Comparable Thickness of Wall and Skin-Layer[1]

G.Sh. Boltachev, N.B. Volkov, S.N. Paranin, and A.V. Spirin

*Institute of Electrophysics UB RAS, 106, Amundsen str., 620016, Ekaterinburg, Russia*
*Phone: 8(3432) 67-87-76, Fax: 8(3432) 67-87-94, E-mail: grey@iep.uran.ru*

**Abstract – The theoretical model of the axially symmetric motion of a conductive shell under pressure of external longitudinal pulsed magnetic field has been developed. The pulsed magnetic field is generated by a helix coil. The program package, which allows modeling both the dynamics of the shell being deformed and the dynamics of an LC-oscillatory circuit, has been worked out. Within this model, the magnetic field diffusion as well as inertial (density) and strength (yield stress) properties of the shell are taken into account. The comparative analysis of behavior of the shells composed of copper, aluminum, and stainless steel, has been carried out. The conditions have been disclosed when expansion of the shell being in external magnetic field is possible due to the diffusion effect.**

## 1. Introduction

As is known, a radial compression of a conductive hollow cylinder occurs when it is placed into a strong external pulsed magnetic field [1–4]. In that case, the processes of magnetic field diffusion, deformation of the shell (the hollow cylinder), and transient processes in the discharging oscillatory circuit generating the magnetic field are interconnected. Thus, the radial compression of the shell is governed by both the pulsed current parameters and the properties of the shell material. Processes of this kind, in particular, take place at the magnetic pulsed compaction of powders [3, 5–7].

Apart from the compression, an induction expansion of the shell can be realized if a solenoid is disposed inside the shell. It has been shown [4] that the expansion of a thin-walled cylinder is even possible when a solenoid is disposed outside the cylinder. If the external magnetic field applied to the outside surface of the shell has a form of triangular pulse with a smooth pulse rise and an abrupt decline then there is nonzero magnetic field inside the shell when the external field is off. At this instant, the internal magnetic pressure can result in the expansion of the shell.

In this study, we investigate the possibility to expand a shell (a metal tube) disposed just inside the solenoid through which a single half-period of sinusoidal current pulse is passed. Such a form of pulsed current can be obtained by discharging capacitor storage on the solenoid with the use of vacuum gap as a switch. The vacuum gap is a diode rectifier as well. The expansion of the metal tube in this way at magnetic pulsed compaction will allow, in particular, extracting the compacted sample from its container after the compaction process safely.

## 2. Theoretical model

Consider coaxial solenoid and conducting shell. The cross section of the system is schematically depicted In Fig. 1. Dynamics of an oscillatory circuit consisted of a capacitor bank with capacity $C$ and a solenoid with $N_s$ turns is described by the equation

$$Ir_e = \frac{q}{C} - l_e \frac{dI}{dt} - N_s \Delta\phi , \quad (1)$$

where $r_e$, $l_e$ are the ohmic resistance and the inductance of the circuit without the solenoid, $q$ is the charge of the capacitor bank, $t$ is the time, $I = -dq/dt$ is the current, $\Delta\phi$ is the potential difference on a turn of the solenoid. All the turns are supposed to be identical. Edge effects will only be taken into account via the parameters $r_e$ and $l_e$. The potential difference $\Delta\phi$ is determined by the self-induction force that is caused by changing the magnetic field flux within the solenoid. Applying the Ohm's law to a single turn or the conducting shell gives the following relations:

$$\Delta\phi = 2\pi\left(R_b \rho_s j_b - R_c \rho_c j_c\right) + \pi\left(R_b^2 - R_c^2\right)\frac{dB_b}{dt} , \quad (2)$$

$$2R_c \rho_c j_c = -R_p^2 \frac{dB_p}{dt} - 2\int_{R_p}^{R_c} \frac{\partial B(r,t)}{\partial t} r\, dr , \quad (3)$$

$$2\rho_c j_p = -R_p \frac{dB_p}{dt} , \quad (4)$$

where $R_a$, $R_b$, $R_c$, and $R_p$ are the outside and inside radiuses of the solenoid and the shell (see Fig. 1);

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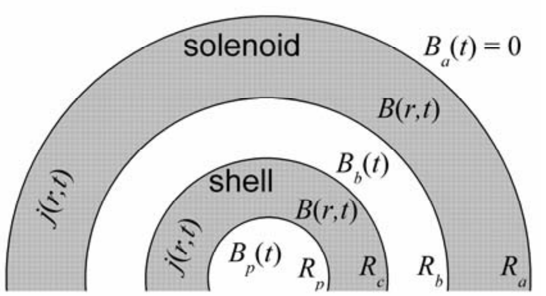


Fig. 1. Schematic representation of the system: cros-section

$j_b(t) = j(R_b,t)$, $j_c(t) = j(R_c,t)$, $j_p(t) = j(R_p,t)$; $B(r, t)$ is the spatially inhomogeneous induction of the magnetic field within the solenoid or the shell, $B_b(t)$ is the induction within the gap between the shell and the solenoid, $B_p(t)$ is the induction inside the shell; $\rho_s$ and $\rho_c$ are the specific resistances of the shell and the solenoid materials, respectively. In the approximation of infinitely long solenoid the relations between the inductions ($B_b$ and $B_p$) and the currents, which pass through the solenoid ($I$) and the shell ($I_c$), can be written as

$$B_b(t) = \frac{\mu_0}{h_{sp}} I(t), \quad I(t) = h_{st} \int_{R_b}^{R_a} j(r,t)dr, \qquad (5)$$

$$B_p(t) = B_b + \frac{\mu_0}{l_c} I_c(t), \quad I_c(t) = l_c \int_{R_p}^{R_c} j(r,t)dr, \quad (6)$$

where $h_{sp}$ is the coil pitch, $h_{st}$ is the turn thickness, $l_c = N_s h_{sp}$ is the shell length, $\mu_0 = 4\pi \cdot 10^{-7}$ H m$^{-1}$. Combination of Eqs. (1), (2), and (5) results in the differential equation

$$\frac{q}{C} - r_e I - (l_e + L_s)\frac{dI}{dt} - 2\pi N_s (R_b \rho_s j_b - R_c \rho_c j_c) = 0, \quad (7)$$

where $L_s = N_s \pi (R_b^2 - R_c^2)\mu_0 / h_{sp}$ is the inductance of the "solenoid + shell" system at the absence of magnetic diffusion, i.e., when a pronounced skin-effect takes place [4]. Initial conditions to Eq. (7) are

$$q(0) = q_0 = CU_0, \qquad \left(\frac{dq}{dt}\right)_{t=0} = I_0 = 0, \qquad (8)$$

where $U_0$ is the initial (charging) voltage of the capacitor bank.

Consider now the radial distributions of the magnetic field within the shell and the solenoid turns, that are needed to calculate the current densities, $j_b(t)$ and $j_c(t)$ in Eq. (7). Since the magnetic fields under consideration are significantly higher than saturation fields, the magnetic permeability of materials is assumed to be similar to that in vacuum, i.e., $\mu = \mu_0$. Then for cylindrical symmetry conditions the magnetic diffusion equation [1, 4], that describes a distribution of a magnetic field, becomes as follows:

$$\frac{\partial B}{\partial t} = \kappa\left(\frac{\partial^2 B}{\partial r^2} + \frac{1}{r}\frac{\partial B}{\partial r}\right), \qquad (9)$$

where the diffusion coefficient $\kappa = (\rho_s h_{sp})/(\mu h_{st})$ for the solenoid and $\kappa = \rho_c/\mu$ for the shell. The boundary conditions for Eq. (9) are: (i) $B_a(t) \equiv 0$ at $r = R_a$ (i.e., the magnetic field outside the solenoid is absent); (ii) and (iii) $B(R_b) = B(R_c) = B_b$, which is determined by the Ampere's circuital law, Eq. (5). As well, Eq. (6) can be the (iv) boundary condition at $r = R_p$, but it is not well behaved, since Eq. (6) is an integral relation with respect to desired function $B(r,t)$. For numerical calculation, it is more appropriate to use a local boundary condition. It can be found by combination Eq. (4) with Eq. (6) in differential form, $j = \mathrm{rot}(B/\mu)$. Thus, the boundary condition (iv) is given by:

$$\left.\frac{\partial B}{\partial r}\right|_{r=R_p} = \frac{\mu R_p}{2\rho_c}\frac{dB_p}{dt}. \qquad (10)$$

Computational solution of diffusion equations (9) for the solenoid and the shell was performed in the framework of the Crank–Nicolson's finite difference scheme of second order, i.e., $O(\tau^2, h^2)$. To keep the calculation accuracy of second order ordinary differential equation (7) was integrated by the implicit one-step method of Adams–Moulton [8] with respect to the function $I(t)$.

### 3. Experiment

To approve the theoretical model and to determine the free parameters $r_e$ and $l_e$, the experimental measurements of circuit parameters at discharging the capacitor storage ($C = 430$ μF) on the solenoid of hardened steel denoted in Russian as 30HGSA ($\rho_s = 210 \times 10^9$ ohm · m) were performed. The solenoid was characterized with parameters: $R_a = 40$ mm, $R_b = 12.85$ mm, $N_s \simeq 21$, $h_{sp} \simeq 6.7$ mm, $h_{sp} \simeq 5.4$ mm. The charging voltages of 5, 10, and 13 kV were used. Within the limits of experiment accuracy the data obtained meets the linear relation between the current and the voltage, $I \sim U_0$, i.e., nonlinear effects concerned with Joule heating are inessential at voltages $U_0 \le 13$ kV. In this connection, Figs. 2 and 3 present the current values normalized on values of the charging voltage. The parameters $l_e$ and $r_e$ are determined from the current time-bases at discharging the storage on the solenoid without shell. The best agreement between theoretical and experimental data is obtained with $l_e = 0.45$ μH and $r_e = 6.0$ milliohm, see Fig. 2.

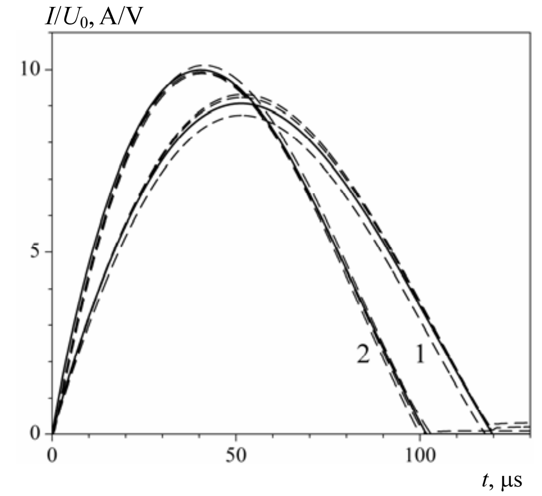


Fig. 2. Comparison between experimental and theoretical data concerning the time-dependence of the current. 1: the solenoid without shell, 2: the "solenoid + shell" system. Solid lines are theoretical ($r_e$ = 6.0 milliohm, $l_e$ = 0.45 μH). Dashed lines are experimental at different charging voltages

At fixed values of parameters $l_e$ and $r_e$ the theoretical model is entirely determined and can be used to calculate the magnetic field diffusion through conductive shells. Correctness of the model is confirmed by the comparison between it and the experimental data on current time-bases in presence of conductive shells inside the solenoid. Fig. 2 presents current time-bases $I(t)$ calculated and measured for copper shells with radial dimensions: $R_c$ = 9.1 mm and $R_p$ = 8.1 mm. Fig. 3 gives the maximum values of the current $I$ for different diameters of the copper shell.

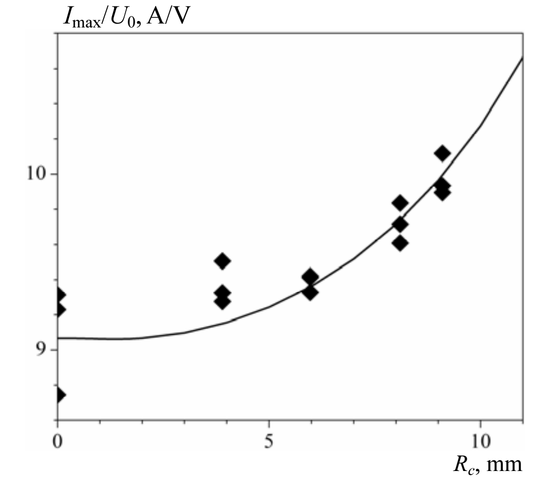


Fig. 3. Maximum value of the current passing through the solenoid as a function of the shell radius $R_c$. The shell thickness is 1.0 mm, the shell material is copper. Line is theoretical curve, symbols are experimental data

## 4. Expansion of the shell

Difference between the magnetic fields inside and outside the conductive shell ($B_p \neq B_b$) results in appearance of radial forces, which aspire to compress or expand the shell. Define the outward force as positive. In that case, the difference of "magnetic pressures" acting on the shell can be written as

$$\Delta p_m = \frac{B_p^2 - B_b^2}{2\mu}. \quad (11)$$

This effort ($\Delta p_m$) is opposed to elastic stresses of the shell. The stresses distribution in the shell corresponds to a solution of the classic Lame problem [5, 9]. In particular, for the strength of elastic-stress deviator, we have

$$\tau = |\Delta p_m| \frac{R_c^2 R_p^2 \sqrt{2}}{r^2 \left(R_c^2 - R_p^2\right)}. \quad (12)$$

Assume that plastic deforming the shell starts when at outside radius, $r = R_b$, the deviator strength has attained the yield stress of material, $\tau_c$. Then the maximal pressure difference $\Delta p_e$, which can be compensated by the elastic stresses of the shell, is

$$\Delta p_e = \frac{\tau_c}{\sqrt{2}} \left(\frac{R_c^2}{R_p^2} - 1\right). \quad (13)$$

The difference of quantities (11) and (13) gives the excess pressure, $\Delta p = \Delta p_m - \Delta p_e$, upon the yield point. Positive $\Delta p$ must lead to the expansion of the shell.

Fig. 4 presents the maximal values of quantity $\Delta p$ calculated for copper shells ($\tau_c \simeq$ 60 MPa) of different diameters and wall thicknesses. It is seen that in the framework of the magnetic pulsed scheme employed the copper shells of 1 mm in thickness cannot be expanded at $U_0$ = 13 kV at the expense of the magnetic field diffusion through shells. However, at the reduction the wall thickness to 0.9 mm the region of shell

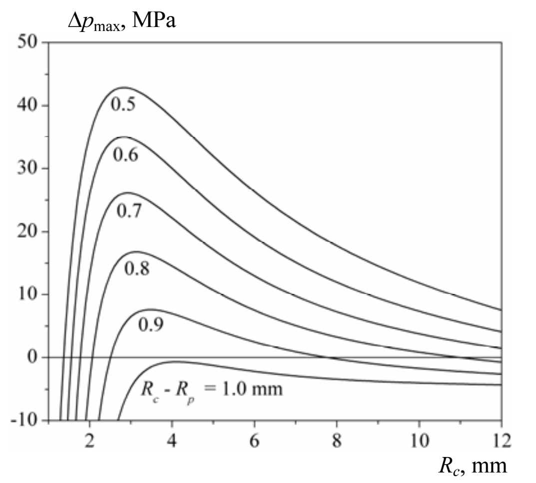


Fig. 4. Maximal value of excess pressure upon the yield point as a function of the shell outside radius $R_c$ at the different wall thicknesses and the fixed charging voltage, $U_0$ = 13 kV

radiuses appears ( $R_c \simeq 2.5 \div 7.5$ ) where such an effect can be realized.

Another way to expand the shell is to increase the charging voltage of the storage. At that, the yield stress $\Delta p_e$ is the same but the magnetic pressure $\Delta p_m$ increases as $U_0^2$ while, at least, the Joule heating can be ignored. However, the amplitudes of magnetic field $B_b$ inside the solenoid at $U_0 = 13$ kV are already of order 25 T. Further increasing of epy charging voltage can lead to the deformation or even destruction of the solenoid.

Besides copper shells, calculations were performed for shells from aluminum ( $\rho_c = 27.7 \cdot 10^{-9}$ ohm · m, $\tau_c \simeq 30$ MPa) and stainless steel denoted in Russian as 12H18N10T ( $\rho_c = 750 \cdot 10^{-9}$ ohm · m, $\tau_c \simeq 216$ MPa). While the expansion of copper shells is possible at wall thicknesses below 1 mm and radiuses of about 3–4 mm (see Fig. 4), the same characteristics for aluminum shells are 1.6 mm and 4.7 mm, respectively. As to steel shells, the quantity $\Delta p_{\max}$ is positive at large radiuses, $R_c \to R_b$, and at wall thicknesses below 1.3 mm. However, the values $\Delta p$ for steel shells are significantly reduced due to the strong magnetic field diffusion and high value of the steel yield stress.

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